Math 170A, Fall 2025 ***HOMEWORK #0*** ***due Thursday, Oct 2***

Questions of this homework will ONLY be graded for completion. You may wish to read chapters 6.1-6.3 of the textbook for Q3 and Q4.

Q1. (2 pts) Find matrices C_1 and C_2 containing independent columns of A_1 and A_2, respectively:

$$A_1 = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 9 & -6 \\ 2 & 6 & -4 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Q2. (2 pts) Read Theorem 1.10 in the textbook, and prove that the following three statements are equivalent.

(b)⇒(a)
take $b = e_i$, $\exists x_i$, s.t. $Ax_i = e_i$ | $B = [x_1\ x_2\ \cdots\ x_m]$, $AB = I$

(a) A is invertible.

(b) For every $b \in \mathbb{R}^m$, there exists exactly one $x \in \mathbb{R}^m$ with $Ax = b$.

(c) The only $x \in \mathbb{R}^m$ with $Ax = 0$ is $x = 0$. Equivalently, the null space $N(A) = \{0\}$.

Q3. (3 pts) Write a MATLAB function with inputs

(c)⇒(b)
Rank(null space) = 0
⇒ Rank(column space) = m
⇒ $\forall b, \exists x, Ax = b$

Assume $Ay = b$
⇒ $A(x-y) = b-b = 0$
⇒ $x = y$

- a matrix A of size $m \times n$,
- a vector x of size $n \times 1$,

and which outputs the product vector Ax (note that the size of this vector should be $m \times 1$).

To get the dimensions of the matrix A, you can use

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[m,n] = size(A);
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similarly to how we got the length of the vector u as

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n = length(u);
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Here the semicolon supresses output.

Your code should check that the sizes of the inputs are right and then do the multiplication using two nested "for" loops.

a) Run your code on $A = rand(7)$ and $x = rand(7, 1)$ (the command "rand" produces a random matrix). Compare the output of your function to the result when you type $A * x$ into MATLAB (the results should be the same!).

b) How many floating point operations (additions and multiplications) does the code use? Find a **formula** in terms of m and n. (Similar to how we found that a vector-vector multiplication takes $2n$ (or $2n-1$, depending on how you count) operations, where n is the vector length.)

Q4. (3 pts)

Write a MATLAB function with inputs

- a matrix A of size $m \times n$,
- a matrix B of size $n \times p$,

and which outputs the product matrix $A \times B$ (note that the size of this matrix should be $m \times p$). Your code should check that the sizes are right and then do the multiplication using three nested "for" loops.

a) Run your code on $A = rand(8, 5)$ and $B = rand(5, 3)$ (the command "rand" produces a random matrix). Compare the output of your function to the result when you type $A * B$ into MATLAB (the results should be the same!).

b) How many floating point operations (additions and multiplications) does the code use? Find a formula in terms of m, n, and p.