Math 170A, Winter 2026 HOMEWORK #1 due Friday, Jan 16

Homework problems that will be graded (Q1 - Q4, 30pts in total).

The first two problems, **Q1** and **Q2**, are meant to help you see why the result of Gaussian Elimination can be obtained by multiplication of A by L^{-1}, where L is the matrix of multipliers. **Q3** proves that the inverse of a upper triangular matrix is also upper triangular. The MATLAB question **Q4** is meant to illustrate the principle "work smarter, not harder". =)

Q1. (8pts) Let n be an integer, and A be a $n \times n$ matrix. Let $a \neq 0$ be a constant.

For a fixed pair of indices (j, i) with $1 \leq i < j \leq n$, we let B_a be the matrix whose entries are all 0, except for the (j, i) entry, which is a (note we set this up so that B_a is strictly lower triangular).

Now let L_a be the $n \times n$ unit lower triangular matrix

$$L_a = I + B_a,$$

where I is the identity matrix of dimension n.

(a) Let $X = L_a A$. Prove that X is the result of the row operation $R_j + aR_i \to R_j$ (adding a times R_i to R_j) on the matrix A.

(b) Show that the inverse of L_a is

$$(L_a)^{-1} = I - B_a.$$

Hint: Think about which row operation you would apply to the matrix X to transform it back into A...

Q2. (6pts) Let $L = \{l_{ij}\}$ be a lower triangular invertible $n \times n$ matrix, and b be a $n \times 1$ vector. Consider solving $Lx = b$ by forward substitution, starting with x_1, which is obtainable from the first equation $l_{11}x_1 = b_1$.

a) Write down equation i in terms of the entries l_{ij} of L, b_i of b, and $x_1, \ldots, x_i$ of x.

b) Assuming that we have already found $x_1, x_2, \ldots, x_{i-1}$, write down the formula for x_i involving the entries l_{ij}, $x_1, \ldots, x_{i-1}$ and b_i.

Q3. (9pts) Let U be an invertible upper triangular $n \times n$ matrix.

a) Let b a vector of length n with $b_i \neq 0$, and $b_{i+1} = b_{i+2} = \ldots = b_n = 0$, for some $1 \leq i \leq n$. Let x be the unique solution to the system $Ux = b$.

By using backward substitution, show that x has the same pattern of zeros as b, that is, $x_i \neq 0$, and $x_{i+1} = x_{i+2} = \ldots = x_n = 0$.

Hint: Start at the end and use "reverse" induction to work your way backward.

b) Prove that U^{-1} is also, like U, upper triangular.

Hint: One possibility is to note that the j-th column x_j of U^{-1} satisfies

$$U\,x_j = e_j, \quad j = 1, \ldots, n$$

where n is the size of the matrix and e_j is the j-th unit vector (jth column of the identity), with a 1 in position j and 0s everywhere else. Then, use a).

Q4. (7pts) Using basic programming ("for" loops and "if" statements), write two MATLAB functions that both take as an input

- $n \times n$ matrix A,
- $n \times n$ matrix B,
- $n \times 1$ vector x.

Check that the inputs are square matrices of the same size. Then, have the first function compute ABx through $(AB)x$ and the second through $A(Bx)$. Then

(a) Take a screenshot of your first function.

(b) Take a screenshot of your second function.

(c) Calculate the number of flops for both approaches, theoretically (in terms of n) and compare: which one is better to use?