PROBLEM SET I

DUE WEDNESDAY, JANUARY 15, 11:59PM

Please neatly write your solutions in complete sentences and label them clearly. To submit your work, log in to Gradescope with your @ucsd.edu email (either directly or through Canvas). Please make sure the scan is clear and prepare early since uploading takes some time.

Problem 1**.** Let $f(x)$ be given by

$$f(x) = \begin{cases} x & x \in \mathbb{Q} \\ x^2 + x & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$$

(a). Show that $f(x)$ is discontinuous when $x \neq 0$.
(b). Show that $f(x)$ is differentiable at $x = 0$. Compute $f'(0)$.

Problem 2. Exercise $1, 2, 4, 5, 6$ from [Rudin] Chapter 5.

Problem 3. Suppose that $f(x)$ is continuous at $x = 0$ and

$$\lim_{x \to 0} \frac{f(2x) - f(x)}{x} = A.$$

Show that $f(x)$ is differentiable at $x = 0$ and $f'(0) = A$.