**Homework due Sunday, December 1st, by 11:59 pm Pacific Time.**

Rudin, Chapter 4 (page 98), problems # 1, 2, 3, 4, 5 (first part only), 6 (Use the metric $d((x_1, y_1), (x_2, y_2)) = d_X(x_1, x_2) + d_Y(y_1, y_2)$), 8, 9, 14.

A. Let $f : [a, b] \to \mathbb{R}$ be a bounded function. For every $x \in [a, b]$ define the function $J_{f,x} : (0, , \infty) \to \mathbb{R}$ by

$$J_{f,x}(r) = \operatorname{diam}\Big(f\Big((x - r, x + r) \cap [a, b]\Big)\Big).$$

(1) Prove that $\lim_{r\to 0^+} J_{f,x}(r)$ exists for every $x \in [a, b]$. Denote this limit by $J_f(x)$.
(2) Prove that $f$ is continuous at $x$ if and only if $J_f(x) = 0$.
(3) Show that for every $\varepsilon > 0$ the set

$$\{x \in [a, b] : J_f(x) \geq \varepsilon\}$$

is a closed set.

The following problems are for your practice, and will not be graded.

(1) Negate the definition of "$\lim_{x\to p} f(x)$".
(2) Negate the definition of "$f$ is continuous at $p$".
(3) Let the notation be as in problem A.
    (a) Show that the set of discontinuities of $f$ is a union of (at most) countably many closed sets.
    (b) Construct (with justification) a function on $\mathbb{R}$ which is discontinuous on $\mathbb{Q}$ and continuous on $\mathbb{Q}^c$. (Hint: recall that $\mathbb{Q}$ is countable.)
    (c) Does there exist any function on $\mathbb{R}$ which is discontinuous on $\mathbb{Q}^c$ and continuous on $\mathbb{Q}$?