Homework 5 due Friday, November 1st, at 11:59 pm Pacific Time.

Rudin, Chapter 2 (page 43), problems #13, 16, 18, 19, 20, 21, 25, 26,

A. Prove that a metric space X is connected if and only if the following holds. The only subsets of X which are both closed and open are $\emptyset$ and X.

The following problems are for your practice, and will not be graded.

(1) Let (X, d) be a set equipped with the discrete metric.
  (a) Show that every subset $A \subset X$ is both open and closed.
  (b) Let A be a nonempty subset of X. Show that A is connected if and only if A has one element.
  (c) Let $A \subset X$. Show that A is compact if and only if A is finite.

(2) Let (X, d) be a metric space, and let $K_1, K_2 \subset X$ be two compact subsets. Suppose $K_1 \cap K_2 = \emptyset$. Prove that there exist open subsets $K_i \subset O_i$, $i = 1, 2$, so that $O_1 \cap O_2 = \emptyset$.