Math 105: Homework 2

Due October 15, 2025

Most questions are from the textbook but have been copied here for your convenience.

1. What is the gcd of $m = 2^2 \cdot 5 \cdot 17 \cdot 19^4$ and $n = 3 \cdot 5^3 \cdot 11^4 \cdot 19$?

2. Show that $\log_{10}(2)$ is irrational.

3. Prove that $\sqrt[3]{3}$ is irrational.

4. Prove that the equation $x^4 - 10x^2 + 1 = 0$ has no rational roots.

5. Prove that $\sqrt{2} + \sqrt{3}$ is a root of $x^4 - 10x^2 + 1$. Conclude that $\sqrt{2} + \sqrt{3}$ is irrational.

6. The Greeks defined the numbers m and n to be **amicable** if

$$\sigma(m) - m = n, \sigma(n) - n = m.$$

   Verify that 220 and 284 are amicable.

7. Prove that if $2^p - 1$ is a prime, then

$$n = 2^{p-1}(2^p - 1)$$

   is a perfect number.

As a reminder, please write clearly and fully explain your solutions. It is OK (and even encouraged) to work with your classmates to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. You are not allowed to search the internet and/or use LLMs to aid you in completing this homework.