Math 105: Homework 1

Due October 08, 2025

1. If a positive integer n is composite, show that there exists a prime number $p \leq \sqrt{n}$ such that p divides n.
2. Let p and q be two consecutive members of the sequence of odd primes

   $$3, 5, 7, 11 \ldots .$$

   Show that $p+q$ factors into at least three (not necessarily distinct) primes. As an example $7 + 11 = 18 = 2 \cdot 3 \cdot 3$.
3. Express the gcd of 17 and 37 as integral linear combination of 17 and 37.
4. Are there integers r and s such that $1841r + 3647s = 1$. Why or Why not?
5. Show that if there are no common prime divisors of a and b, then $(a, b) = 1$.
6. Show that if p is a prime integer, then either $(a, p) = 1$ or $(a, p) = p$.

As a reminder, please write clearly and fully explain your solutions. It is OK (and even encouraged) to work with your classmates to solve the problems, but if you do so, you should write your solutions up separately. Copying solutions from your peers or a solutions manual will be deemed academic misconduct. You are not allowed to search the internet and/or use LLMs to aid you in completing this homework.