Homework due Friday, November 8, 11:59pm

(1) (a) Solve the quadratic congruences

$$2x^2 + 3x + 4 \equiv 0 \pmod{7}$$

and

$$x^2 + 7x + 11 \equiv 0 \pmod{19}.$$

(b) Let p be an odd prime. Show that the number of solutions to the equation $ax^2 + bx + c \equiv 0 \pmod{p}$ is $1 + \left(\frac{b^2-4ac}{p}\right)$.

(2) Compute $\left(\frac{79}{127}\right)$, $\left(\frac{47}{101}\right)$, and $\left(\frac{103}{137}\right)$ by hand.

(3) Let p be an odd prime, and let $a, b \in \mathbb{Z}$ with $p \nmid a$. Show that

$$\sum_{x=0}^{p-1} \left(\frac{ax+b}{p}\right) = 0.$$

(4) Let n be a positive integer, and suppose $p \mid n^4 - n^2 + 1$. Show that $p \equiv 1 \pmod{12}$. Hint: use question 1 and the fact that $(n^2-1)^2 \equiv -n^2 \pmod{p}$.

(5) Show that if $p = a^2 + b^2$ where $a, b \in \mathbb{Z}$ and a is odd, then $\left(\frac{a}{p}\right) = 1$.

(6) Characterize the primes p such that

(a) 10 is a quadratic residue $\pmod{p}$

(b) 21 is a quadratic residue $\pmod{p}$