

Math 105: Homework 7

Due November 24, 2025

Most questions are from the textbook but have been copied here for your convenience.

1. Assume $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$ is the prime factorization of n . Let a be an integer coprime to n . Let r be the least common multiple of $b_1 := \phi(p_1^{a_1}), b_2 := \phi(p_2^{a_2}), \dots$, and $b_r := \phi(p_r^{a_r})$. Show that $a^r \equiv 1 \pmod{n}$. This generalizes Problem 2 from Homework 5.
2. Read Sections 1.1, 3.1, and 5.1, and write “I affirm that I have read Sections 1.1, 3.1, and 5.1” as your “solution” to this problem.
3. Find all positive integer solutions to $x^2 + 12 = y^4$.
4. Solve $x^2 - 7y^2 = 3z^2$.
5. Find all solutions to the equation $x^2 + 3y^2 = z^2$ with x, y , and z positive integer and y odd.